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Stats 209 — Mrs patrick

Probability Distribution

It is divided into 2

- Discrete probability distribution:

When a random variable is drawn from a discrete distribution. The distribution is known as probability mass function. A pmf is used to describe the probability distribution of a discrete random variable.

Let x be a discrete random variable, the PMF is denoted by "$P(x)$" which satisfies:

a) $0 \leq P(x) \leq 1$

b) $\sum P(x) = 1$ (over all possible values of x)

Some Common discrete distributions and their PMF

1. Bernoulli Distribution

Definition:

A random variable X is Bernoulli(p) if it takes values 0 and 1 with Pmf:

$$P(X=x) = p^x(1-p)^{1-x} \quad x = 0, 1$$

$$E(X) = p$$

$Var(X) = P(1-P)$.

Example 1

Let $X \sim Ber(0.6)$ find $P(X=1)$, $E(X)$ and $Var(X)$

Solution

Since $P = 0.6$

then $P(X=1) = 0.6$ - Success

therefore $P(X=0) = 1 - \text{Success} = 1 - 0.6 = 0.4$ (failure)

$E(X) = P = 0.6$

$Var(X) = P(1-P) = 0.6(0.4$

$= 0.24$

Example 2

A new mobile app has 70% chance of being demanded Let $X=1$ if downloaded and $X=0$ otherwise. find the distribution of X, $E(X)$ and $Var(X)$

Solution

$X \sim$ Bernoulli (0.7)

$E(X) = 0.7$

$Var(X) = 0.7 \times 0.3 = 0.21$

2. Binomial Distribution

A random variable X in $Bin(n,p)$ with the pmf

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x} \quad x = 0, 1, 2 \cdots n$$

the binomial distribution models the number of successes in n independent trials.

$$E(X) = np$$

$$Var(X) = np(1-p)$$

Example:

Let $X \sim Bin(5, 0.4)$ find $P(X=2)$, $E(X)$, $Var(X)$

Solution

$$P(X=2) = \binom{5}{2} (0.4)^2 (0.6)^3 = 10 \times 0.16 \times 0.216 = 0.3456$$

$$E(X) = np = 5 \times 0.4 = 2$$

$$Var(X) = np(1-p) = 5 \times 0.4 \times 0.6 = 12$$

Example 2:
In a sample of 6 bulbs each has a 10% chance of being defective let $Y \sim Bin(6, 0.1)$. find $P(Y=2)$, $E(Y)$, $Var(Y)$

Solution

$$P(Y=2) = \binom{6}{2} (0.1)^2 (0.9)^4$$
$$= 15 \times 0.01 \times 0.6561$$
$$\approx 0.0984$$

$$E(Y) = 6 \times 0.1 = 0.6$$
$$Var(Y) = 6 \times 0.1 = 0.6 \times 6.9 = 0.54$$

~~Var(~~

Poisson Distribution

The poisson distribution models the number of events occuring in a fixed interval of time or space, given that the events occur with a known constant mean rate and independently of the time since the last event occured.

If random variable $X \sim$ poisson (λ) with PMF

Mean $E(x) = \lambda$

Variance $Var(x) = \lambda$

$$P(X=x) = \frac{e^{-\lambda} \lambda^{x}}{x!} \quad x = 0, 1, 2 \ldots\ldots$$

Example

Suppose a Call Centre recieves an average of $\lambda = 3$ calls per hour. What is the probability that they recieve exactly 4 calls in an hour?

Solution

$$P(X=4) = \frac{e^{-3} \times 3^{4}}{4!} = \frac{e^{-3} \times 81}{24} \approx 1.68$$

Probability Distribution function (Pdf)

for a continuous random variable, we use p.d.f

1. $f(x) \geq 0$
2. $\int_{-\infty}^{\infty} f(x)\, dx = 1$

(1) Exponential distribution

Pdf · $P(X=x) = f(x) = \begin{cases} \lambda e^{-\lambda x} & x>0 \\ 0 & \text{otherwise.} \end{cases}$

$E(x) = \frac{1}{\lambda}$

$Var(x) = \sigma^2 = \frac{1}{\lambda^2}$

Example:

The lifetime (10 years) of a machine follows an exponential distribution · with parameter $\lambda = 0.2$

Solution

a) find $P(X>5)$

b) find the mean lifetime

Solution

$f(x) = \lambda e^{-\lambda x}, \quad x>0$

$f(x) = 0.2 e^{-0.2x}$

a) $P(x>5) = \int_5^{\infty} 0.2 e^{-0.2x} dx$

b)

$$= 0.2 \int_5^{\infty} e^{-0.2x} \, dx$$

$$\frac{0.2 \times -1}{0.2} \left[e^{-0.2x} \right]_5^{\infty}$$

$$= -1 \left[e^{-0.2x} \right]_5^{\infty}$$

$$= -1 \left[e^{-0.2(\infty)} - e^{-0.2(5)} \right]$$

$$= -1 \left[e^{-\infty} - e^{-1} \right]$$

$$= -1 \left[\frac{1}{e^{\infty}} - \frac{1}{e^{1}} \right]$$

$$= -1 \left[\frac{1}{\infty} - \frac{1}{e} \right]$$

$$= -1\left[0 - e^{-1}\right]$$

$$= -1\left[-e^{-1}\right]$$

$$= e^{-1}$$

$$\approx 0.368$$

Mean lifetime $\Rightarrow E(X) = \frac{1}{\lambda}$

$$= \frac{1}{0.2}$$

$$= 5 \text{ years.}$$