PROBABILITY & Random Variables

Probability is simply a game of chance. The application of probability is evident in most areas of human endeavor. for example:

- The chance of an accident occuring on a road
- Probability of getting a head when a coin is tossed.
- Chance of a top politician winning an election.

Such random efforts can easily be predicted within certainty. Therefore we may be able accept the degree of uncertainty. In any given situations and this is done mathematically using probability.

The concept of Probability is necessary when dealing with events that are not predictable. The mostly used concept of probability is the relative frequency because it deals with occurrence of event in a long series of trial, this concept somehow invalidate the classical probability because of its limitation.

PROBABILITY OF EVENTS

EVENT ⇒ Any subset of the sample space.

Example: If A is an event, n(A) is the numbers of sample point that belongs to event A. It is either you fail / pass.

The probability of an event is a measure of likelihood of that event occuring. If an experiment has a finite number which are equally likely, then, the probability of that event e.g Event A will occur is given by. . . . . Therefore the probability of A = numbers of ways A can occur all over total possible outcome

$$P(A) = \frac{\text{No of ways}}{\text{Total outcome}}$$

E.g ACS 209 Exam = {Fail, Pass}

$$P(\text{Fail}) = \frac{1}{2}$$

{A, B, C}

{A, B, C, D, E, F} = $P(A) = \frac{1}{6}$

* Experiment ⇒ any process that yields a result or an observation is an experiment. for example: ACS 209 exam = {fail, pass}

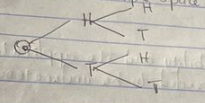
* Outcome ⇒ This is a particular result of an experiment

* Sample space ⇒ (n) is the set of all possible outcome of an experiment.

* Sample point ⇒ this is the individual outcome in a sample space. If sample point correspond to a single outcome of an experiment

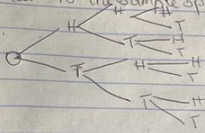
Examples

1. A die is tossed once and the outcomes could be any of these
   $S = \{1, 2, 3, 4, 5, 6\}$
2. A coin is tossed twice and the outcome for each toss is recorded. What is the sample space?



$S = \{HH, HT, TH, TT\}$

3. If the coin is tossed thrice and the outcome is recorded. What is the sample space



$S = \{HHH, HHT, HTH, HTT, THH, TTT, TTH, TTT\}$

Two dice are rolled, and the sum of the numbers are observed and recorded, what is the sample space?

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$= \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

LAWS OF PROBABILITY

understanding the concept of probability an
laws of probability are essentially needed t

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

LAWS OF PROBABILITY

To understanding the concept of probability and its application the laws of probability are essentially needed to be understood. A probability is always numerical between 0 and 1

1. If A is an event that can happen. $\therefore P_r(A) \geq 0$ or $P(A) \leq 1$

That is $0 \leq P(A) \geq 1$

2. The summation of the probability of all the outcome of the experiment is always equal to 1.

$$\sum_{\text{all the outcome of experiment}} P(A) = 1$$

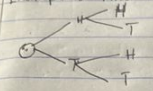
Summation
For discrete $\sum$
<< continous $\int$

Note: the probability of any event A is always greater than or equal to 0 ... which means the probability of an event cannot be negative, at the same time, the probability of any event is ≤ 1, that is no probability is greather than 1

2 types of event
1. mutually exclusive
2. Independent

ii) The second remark states that if we add up the probability of each of the sample point in the sample space, the total probability must be equal to 1. That is to say, the sum of the total sample space of the probability must be 1

Example: Find the sample space of tossing 2 coins.



$S = \{HH, HT, TH, TT\}$

For 3 coins

	HH	HT	TH	TT
H	HHH	HHT	HTH	HTT
T	THH	THT	TTH	TTT

The probability that a Head will appear

$\{HT, TH\}$ HT or TH
HT + TH

$P\{a\ head\} = \frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2} = 0.5$

Note: the result of T from the experiment is between $0 < 0.5 < 1$

$S = \{HH\ HT\ TH\ TT\}$

$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$

Mutually Exclusive Event

Those are events defined in such a way that the occurence one of the event will not/or ~~exclude~~ exclude the occurence of t her event.

2) Figure 5 appearing twice in a die throw is not possible.
Example:
Supposing we have two event A and B and they are mutually exclusive event.
Eg If Event A is head of throwing a coin and event B is the tail of throwing a coin.

P(A and B) = mutually exclusive (two or more event cannot happen at the same time)
P(A ∩ B) = ∅ (intersection) or empty set

Example: Consider an experiment, in which two dice are tossed. 3 events are defined.

1. Event A: the sum of the numbers on the 2 dice is 6
2. Event B: the sum of the two die is 8
3. Event C: each of the two dice shows the same number

X_1/X_2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Event A & B are mutually exclusive because the sum of the 2 dice can't be both 5 and 8 at the same time. It is clearly seen in the table that event A & B do not intercept.