MATHEMATICS OF FINANCE

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LEARNING OUTCOMES

1. To understand the basic concepts of Mathematics of Finance
2. To understand the difference between simple interest and compounding interest
3. To evaluate the present value and future value of a lump sum
4. To evaluate the present value and future value of an Annuity
5. To apply the concepts to solve some business related problems using sinking fund and loan amortization

CONCEPT OF MATHEMATICS OF FINANCE

- A Naira expected sometimes in the future is not equivalent to a Naira today because of time value of money. Interest is the payment made for the use of money. The concept of time value of money forms the basis of mathematics of finance principles such as simple interest, compounding interest, present values and future value of lump sum and Annuity

DIFFERENCE BETWEEN SIMPLE INTEREST AND COMPOUNDING INTEREST

- SIMPLE INTEREST
- This is the interest earned on an original amount invested or principal. Mathematically, it is defined as
- $SI_T = P \times T \times R$
- Where SI_T = simple interest at the end of year T
- P = Principal or original amount invested
- T = Time in years
- R = Rate in percentage per year

DIFFERENCE BETWEEN SIMPLE INTEREST AND COMPOUNDING INTEREST

- Example 1: Betty invests N100,000 for 3 years at a simple interest of 10% per year. Calculate the total interest earned at the end of three years.
- SOLUTION 1: $SI_T = P \times T \times R$
- Where SI_T = simple interest at the end of year T
- P = Principal or original amount invested =N100,000
- T = Time in years = 3
- R = Rate in percentage per year = 0.1

DIFFERENCE BETWEEN SIMPLE INTEREST AND COMPOUNDING INTEREST

- SI_3 = 100,000 x 3 x 0.1
- SI_3 = N30,000

Example 2: assuming betty invests the money for 4 years and 5 years respectively, determine the simple interest at the of 4years and 5years respectively.

- Solution 2: SI_4 = 100,000 x 4 x 0.1
- SI_4 = N40,000
- SI_5 = 100,000 x 5 x 0.1
- SI_5 = N50,000

DIFFERENCE BETWEEN SIMPLE INTEREST AND COMPOUNDING INTEREST

- COMMENT:
- 1. The annual simple interest (N10,000) remain constant from year to year i.e SI_3 = N30,000, SI_4 = N40,000, and SI_5 = N50,000. The difference from each year remain N10,000
- 2. Principal or original amount invested (N100,000) remain constant from year to year

DIFFERENCE BETWEEN SIMPLE INTEREST AND COMPOUNDING INTEREST

- COMPOUDING INTEREST: This is the interest earned on an original amount invested or principal plus previously earned interest. Interest is added to the principal as we earn it during any year.
- Example 3: Betty invests N100,000 for 3 years at 10% annual compounding interest rate. Calculate the total interest earned at the end of three years.

DIFFERENCE BETWEEN SIMPLE INTEREST AND COMPOUNDING INTEREST

Solution 3

YEAR	PRINCIPAL OR AMOUNT AT THE BEGINNING	COMPOUNDING INTEREST @10%	AMOUNT AT THE END
1	100,000	10,000	110,000
2	110,000	11,000	121,000
3	121,000	12,100	133,100

DIFFERENCE BETWEEN SIMPLE INTEREST AND COMPOUNDING INTEREST

- The total interest earned = N10,000 + N11,000 + N12,100 = N33,100

COMMENT:

1.The annual compounding interest rate increases from year to year i.e CI_1 = N10,000, CI_2 = N11,000, and CI_3 = N12,100.

2. Principal or original amount invested increases from year to year

FUTURE VALUE OF A LUMP SUM

- A Lump sum is defined as a SINGLE amount of money invested for a specified period of time at specified annual compounding interest rate.
- The future value of a lump sum is determined as shown below:
- $FV = PV(1 + i)^n$
- WHERE
- FV = Future Value of a lump sum
- PV = Present value of a lump sum or Principal or Single amount invested
- i -= Annual compounding interest rate
- n = number of years

FUTURE VALUE OF A LUMP SUM

Example 4: Calculate the future value of Betty investment of N100,000 for 3 years at 10% annual compounding interest rate.

Solution 4: $FV = PV(1 + i)^n$

$PV = N100,000$

$i = 0.1$

$n = 3$

$FV = 100,000(1 + 0.1)^3$

$FV = 100,000(1.1)^3$

$FV = N133,100$

FUTURE VALUE OF A LUMP SUM

Example 4: What will be the total receivable from an investment of N800,000 mace for 4 and half years at 12% annual compounding interest rate.

Solution 4: $FV = PV(1 + i)^n$

$PV = N800,000$

$i = 0.12$

$n = 4.5$

$FV = 800,000(1 + 0.12)^{4.5}$

$FV = 800,000(1.12)^{4.5}$

$FV = N1,332,205$

FUTURE VALUE OF A LUMP SUM

Example 5: How long will it take N100,000 to accumulate to N133,100 at 10% annual compounding interest rate.

Solution 5: $FV = PV(1 + i)^n$

$PV = N100,000$

$FV = N133,100$

$i = 0.1$

$133,100 = 100,000(1 + 0.1)^n$

$133,100 = 100,000(1.1)^n$

Divide both sides by 100,000

FUTURE VALUE OF A LUMP SUM

- $1.331 = (1.1)^n$

Obtained the log to base 10 of both sides

- $n = \frac{\log_{10} 1.331}{\log_{10} 1.1}$

$n = 3\text{years}$

FUTURE VALUE OF A LUMP SUM

Example 6:What interest rate compounding annually will make N100,000 to accumulate to N133,100 for 3 years.

Solution 6: $FV = PV(1 + i)^n$

$PV = N100,000$

$FV = N133,100$

$n = 3$

$133,100 = 100,000(1 + i)^3$

$133,100 = 100,000(1 + i)^3$

Divide both sides by 100,000

FUTURE VALUE OF A LUMP SUM

- $1.331 = (1 + i)^3$

Obtained the cube roots of both sides

- $(1.331)^{0.3333} = 1 + i$
- $i = 1.1 – 1$
- $i = 0.1$ or 10%

PRESENT VALUE OF A LUMP SUM

- A Present value of a lump sum is the worth of an amount to be received in future as at now.
- The formula for a present value of a lump sum is just a reciprocal of future value of a lump sum.

PRESENT VALUE OF A LUMP SUM

Example 7:Find the present value of N133,100 to be paid in 3 years at 10% annual compounding interest.

Solution 7: $FV = PV(1 + i)^n$

$FV = N133,100$

$n = 3$

$i = 0.1$

$133,100 = PV(1 + O.1)^3$

$133,100 = PV(1.1)^3$

PRESENT VALUE OF A LUMP SUM

- 133,100 = PV (1.331)
- PV = $\frac{133,100}{1.331}$
- PV = N100,000

MULTI PERIOD COMPOUNDING

- If the interest is compounding more than once in a year, the formula for the future value of a lump sum would be adjusted as shown below:
- $FV = PV(1 + i/m)^{n \times m}$
- WHERE
- FV = Future Value of a lump sum
- PV = Present value of a lump sum or Principal or Single amount invested
- i -= Annual compounding interest rate
- n = number of years
- m = number of times interest is compounding in a year

MULTI PERIOD COMPOUNDING

- For instance
- Semi-annually m = 2
- Every four month m = 3
- Quarterly (every three months) m = 4
- Twice in a quarter m = 8
- Monthly m = 12
- Twice in a month m = 24

MULTI PERIOD COMPOUNDING

Example 8: Assuming N100,000 is paid in a fixed deposit account with a bank that offers the depositor 24% interest per annum payable twice in a quarter. Calculate the compounding value of the investment if it is made for 3 years.

- Solution 8: $FV = PV(1 + i/m)^{n \times m}$
- WHERE
- PV = N100,000
- i -= 0.24
- n = 3
- m = 8

MULTI PERIOD COMPOUNDING

- FV = $100,000(1 + 0.24/8)^{3 \times 8}$
-

$= 100,000(1.03)^{24}$

$FV = N203,279.41$

CONTINOUS COMPOUNDING

- For continuous compounding,
- $FV = PVe^{i \times n}$
- where
- FV = Future Value of a lump sum

  PV = Present value of a lump sum or Principal
- i -= Annual compounding interest rate
- n = number of years
- e = 2.71828 = base of natural lograthm

CONTINOUS COMPOUNDING

- Example 9: Assuming N250,000 is paid in a fixed deposit account with a bank that offers the depositor 17.5% interest per annum. Calculate the compounding value of the investment if it is made for 2 years and interest is compounding continuously.
- Solution 9: $FV = PVe^{i x n}$
- where

  PV = N250,000
- i -= 0.175
- n = 2
- e = 2.71828
-

CONTINOUS COMPOUNDING

- FV = $250{,}000e^{0.175 \times 2}$
-
- $= 250{,}000e^{0.35}$
- = N354,766.80

ANNUITY

An annuity can be defined as a series of equal amount payments or receipts over some periods with compounding interest on the payments or receipts.

There are three types of annuity

1. Ordinary Annuity is a series of equal amount payments or receipts that occur at the end of each period involved.
2. Annuity due is a series of equal amount payments or receipts that occur at the beginning of each period involved.
3. Perpetual Annuity is a constant income stream made up of regularly recurring identical payments which are paid out to a recipient from a lump sum of capital.

FUTURE VALUE OF ORDINARY ANNUITY

DEFINITION: The future value of an ordinary annuity is the sum of all payments, and the compound interest accumulated on each .Note that all the payments made in an ordinary annuity must be paid at the end of each periodic interval. The formula for future value of ordinary annuity is shown on the next slide.

FUTURE VALUE OF ORDINARY ANNUITY

$$FV = A\frac{\{(1 + i)^n - 1\}}{i}$$

- Where;
- A = Annuity per period
- n = number of years
- i = annual compounding interest rate
- FV = Future value of ordinary annuity

FUTURE VALUE OF ORDINARY ANNUITY

Example 10; Raman deposited N20,000 into an account at the end of each year for 3 years at the 20% annual compounding interest rate. Determine the future value.

SOLUTION 10

$$FV = A\left(\frac{(1+i)^n - 1}{i}\right)$$

Year	Amount at Beginning	Annual Interest@20%	Annual saving	Amount at END
1	-	-	20,000	20,000
2	20,000	4,000	20,000	44,000
3	44,000	8,800	20,000	72,800

FUTURE VALUE OF ORDINARY ANNUITY

A = N20,000 n = 3 i = 0.2

$$FV = 20{,}000 \left\{ \frac{(1 + 0.2)^3 - 1}{0.2} \right\}$$

$$= 20{,}000 \left\{ \frac{(1.2)^3 - 1}{0.2} \right\}$$

= N72,800

PRESENT VALUE OF ORDINARY ANNUITY

DEFINITION: The present value of an ordinary annuity is the amount that would have to be invested today at a certain compound interest to enable the investor to receive the series of future payment over a specific period. The formula for present value of ordinary annuity is shown on the next slide.

PRESENT VALUE OF ORDINARY ANNUITY

$$PV = A\frac{\{ 1 - (1 + i)^{-n} \}}{i}$$

- Where;
- A = Annuity per period
- n = number of years
- i = annual compounding interest rate
- PV = Present value of ordinary annuity

PRESENT VALUE OF ORDINARY ANNUITY

Example11: Assume that Joshua has obligation of N120,000 that he must repay at the end of each of the next three years. Determine the present value of his investment today at 12% annual compounding interest rate to accomplish his desire.

Solution 11:

A = 120,000

n = 3

i = 0.12

PV = ?

PRESENT VALUE OF ORDINARY ANNUITY

$$PV = A\frac{\{ 1 - (1 + i)^{-n} \}}{i}$$

$$= 120{,}000\frac{\{ 1 - (1 + 0.12)^{-3} \}}{0.12}$$

$$= 120{,}000\frac{\{ 1 - (1.12)^{-3} \}}{0.12}$$

$$= N288{,}219.75$$

Year	Amount at Beginning	Annual Interest@12%	Annual repayment	Amount at END
1	288,219.70	34,586.37	120,000	202,806.12
2	202,806.12	24,336.73	120,000	107,142.85
3	107,142.85	12,857.14	120,000	0

FUTURE VALUE OF ANNUITY DUE

DEFINITION: The future value of an ordinary annuity is the sum of all payments, and the compound interest accumulated on each .Note that all the payments made in an annuity due must be paid at the beginning of each periodic interval. The formula for future value of annuity due is shown on the next slide.

FUTURE VALUE OF ANNUITY DUE

$$FV = A\frac{\{(1 + i)^{n+1} - (1 + i)\}}{i}$$

- Where;
- A = Annuity per period
- n = number of years
- i = annual compounding interest rate
- FV = Future value of annuity due

FUTURE VALUE OF ANNUITY DUE

Example 12: Supposing in Example 10, Raman has deposited the N20,000 into the saving plan at the beginning of three consecutive years, with each payment earning 20% interest compounded annually . Calculate the future value of the annuity.

SOLUTION 12: $FV = A\frac{\{(1 + i)^{n+1} - (1 + i)\}}{I}$

A = N20,000

i = 0.2

n = 3

FV = ?

$= \frac{20{,}000\ \{ (1 + 0.2)^{3+1} - (1 + 0.2)\}}{0.2}$

$= \frac{20{,}000\ \{ (1.2)^{4} - (1.2)\}}{0.2}$

FUTURE VALUE OF ANNUITY DUE

FV = N87,360

PRESENT VALUE OF ANNUITY DUE

DEFINITION: The present value of an annuity due is the amount that would have to be invested today at a certain compound interest rate to enable the investor to receive the series of future payment from now over a specific period. The formula for present value of annuity due is shown on the next slide.

PRESENT VALUE OF ANNUITY DUE

$$PV = A\left\{\frac{(1 + i) - (1 + i)^{-n + 1}}{i}\right\}$$

- Where;
- A = Annuity per period
- n = number of years
- i = annual compounding interest rate
- PV = Future value of annuity due

PRESENT VALUE OF ANNUITY DUE

Example 13: supposing in Example 11, Joshua must repay the N120,000 obligation at the beginning of each of the next three years. Joshua want to know how much it would have to invest today to repay each obligation if the amount invested earn 12% compounding annually.

Solution 13: $PV = A\frac{\{(1 + i) - (1 + i)^{-n + 1}\}}{i}$

- Where;
- A = N120,000
- n = 3
- i = 0.12

PRESENT VALUE OF ANNUITY DUE

$$PV = 120{,}000\left\{\frac{(1 + 0.12) - (1 + 0.12)^{-3 + 1}}{0.12}\right\}$$

$$= 120{,}000\left\{\frac{(1.12) - (1.12)^{-2}}{0.12}\right\}$$

= N322,806.10

PRESENT VALUE OF PERPETUAL ANNUITY

DEFINITION: A perpetual annuity is a series of equal periodic payments or receipts expected indefinitely. The formula for perpetual annuity is giving below

$$PV = \frac{A}{I}$$

Where

A = Annuity per period

i = Annual compounding interest rate

PRESENT VALUE OF PERPETUAL ANNUITY

Example 14: Chukwu expects a perpetual sum of N250,000 annually from the estate of his deceased uncle. What is the present value of this annuity if Chukwu can invest at the rate of 20% per annum.

Solution 15: $PV = \frac{A}{i}$

Where

A = N250,000

i = 0.2

PRESENT VALUE OF PERPETUAL ANNUITY

$$PV = \frac{A}{i}$$

$$= \frac{250{,}000}{0.2}$$

$$= N1{,}250{,}000$$

IMPORTANT HINTS

1. FOR **300 LEVEL ACCOUNTING AND BUSINESS ADMINISTRATION STUDENTS, PRACTICES SOME QUESTIONS FROM PROBLEM AND SOLUTION IN FINANCIAL MANAGEMENT BY DR SHIRO ABASS, CHAPTER 2, QUESTIONS 1 TO 40.**